Our earliest work on nonlinear wave propagation in rock involved a series of experiments suggested by the theory extant at that time (Johnson et al., 1987). This theory, which we term the traditional theory, is a Landau theory (Landau and Lifshitz, 1986). It was borrowed from solid state physic/acoustics. The traditional theory predicts behavior in qualitative accord with some experimental observations. To provide a partial explanation for our choice of experiments, to have in hand the elements of the traditional theory, and to offer in contrast to the theory introduced below, we sketch the traditional theory here (see, e.g., McCall, 1993).

The empirical and widely known fact is that the velocity of sound in rock is a sensitive function of pressure. The pressure is in turn related to the strain field. One could sensibly write

$$c^2 = c_o^2 \left[1 + \beta \left(\frac{\partial u}{\partial x}\right) + \delta \left(\frac{\partial u}{\partial x}\right)^2 + \ldots \right], \qquad (1)$$

where c is the wave speed, c_o is the wave speed at zero strain, $\left(\frac{\partial u}{\partial x}\right)$ is the strain, and β and δ are nonlinear parameters that characterize the cubic and quartic anharmonicities . [The Landau theory arrives at the functional equivalent of Eq.(1) by employing an elastic energy that is a function of the stain field that is invariant with respect to the symmetry of the rock.] When a strain dependent velocity is incorporated into the wave equation for the displacement field a variety of phenomena are predicted. The most evident of these is the property of harmonic generation. That is, when a monochromatic wave is broadcast into a rock, the nonlinearity in the rock generates a spectrum of harmonics. The treatment of harmonic generation, using the traditional theory, leads to a quantitative description to be tested in order to establish the correctness of the fundamental model (i.e. the Landau theory). Our first experiments were an attempt to verify the simplest predictions of the traditional theory.

Semi-quantitative agreement with these predictions has been found. [For example the amplitude at 2ω is predicted to increase as $(\mathbf{k}U)^2$ and found to increase approximately as $(\mathbf{k}U)^{1.6}$ where $\mathbf{k}$ is wavenumber and U is the source displacement amplitude.] However, from the vantage point afforded by numerous experiments on rock (involving broad strain and stress intervals, a broad frequency range) a more complex picture emerges. Observations show that a great deal is going on that is not adequately described in the traditional theory (see e.g., Johnson et al., 1995 and Guyer et al., 1995). The picture that is emerging from studies on rock is that they have important properties endemic to their nature that can be observed by their nonlinear response, but that cannot be described by a traditional theoretical approach. In the following, we introduce evidence for some of the unusual behavior and sketch the new theoretical paradigm.

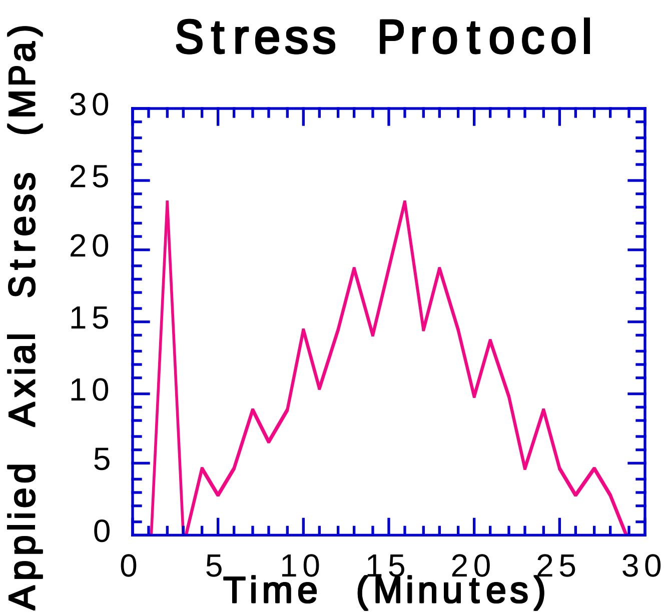
Hysteresis, discrete memory, and nonlinearity.

Introduction. The hysteretic nature of the elastic response of rocks is a behavior having a long and venerable history. For example, the work of Gardner and collaborators (Gardner et al., 1966), in understanding the influence of effective pressure on the equation of state, called particular attention to the importance of "the path to a stress state". Holcomb (1978;1981), in an impressive set of experiments, documented both hysteresis and the remarkable property of discrete memory (also end point memory). It is clear from work this work that the hysteretic behavior of the elastic response provides a strong clue to the microscopic structure of rock, i.e. to the nature of the compliant portion of the material: grain to grain contacts, cracks, contained fluids, etc. It is the behavior of the compliant component of the material that determines the linear/nonlinear response of the rock in a quasi-static measurement, in wave propagation, etc. (see e.g., Gist, 1993). The microscopic structure in rocks has added importance in the context of developing a connection between elastic response and fluid transport.

Hysteresis, discrete memory, nonlinearity can be regarded as a nuisance when attempting a simple characterization of the elastic properties of rock. This is fundamentally because there is no theoretical paradigm to help sort out which velocity of sound or modulus is the "correct" one. Thus it has been standard practice to get eliminate hysteresis or find procedures that mitigate its influence. In doing so one does away with what is potentially of great importance.

A new theoretical paradigm for the description of rock elasticity has been developed at Los Alamos. The properties of hysteresis and discrete memory are used to advantage because we have the means to understand their source and consequences. The paradigm applies data from a rock to construct the fundamental theoretical model for the description of the elastic properties of the rock. Let us illustrate hysteresis and discrete memory seen in a static test on sandstone. We will then sketch the elements of the new paradigm that describes these properties and other elastic properties of rock.

Figure 2a,b illustrates the (a) stress-time history (stress protocol) and the (b) corresponding stress-strain response for a Berea sandstone sample. Results are from a uniaxial static test. Hysteresis is illustrated by the different stress-strain paths of the outer loops in the lower figure. Note that the stress-strain relation is nonlinear along both loops. The multivalued nature of the curves and the fact that they are nonlinear implies that the derivative of stress-strain (the modulus) is also nonlinear and hysteretic. The stress protocol has a number of smaller cyclic excursions in order to explore the effect of smaller stress-strain deviations and to illustrate the property of discrete memory. A rock with discrete memory remembers its elastic state. For example; (1) the rock is taken to a prescribed initial stress state, (2) the stress is slightly relieved, (3) the stress is returned to that of the initial state. The strain in the rock will return to the strain level of the initial state. Further increase of the stress, beyond that of the initial state, leads to the same sequence of strains as those that would have occurred had the cyclic excursion, steps (2) and (3), not occurred. In essence, the rock "remembers" the strain level of the initial state and the stress-strain trajectory it was on.



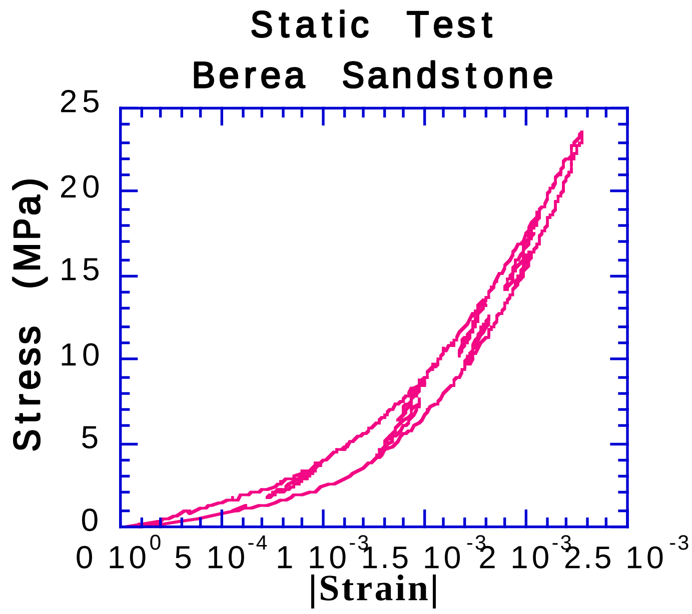


Figure 2. Top: pressure (stress) protocol for result shown in bottom plot. Bottom: static stress-strain data for Berea sandstone illustrating a nonlinear equation of state: hysteresis, and point memory (from Hilbert et al., 1994).

The significant points we note from this experiment are, (1) the stress-strain relation is nonlinear, (2) the stress-strain relation and the modulus-strain relation are hysteretic, (3) the stress-strain relation has discrete memory, and (4) the modulus is stress path dependent. Hysteresis and discrete memory are manifestations of nonlinearity, and vice versa. [Recently Boitnott (1993) and Pyrak-Nolte et. al. (1990) following Scholz and co-workers (Scholz 1979) looked at the elastic properties of joints and found them to have more extreme hysteresis (with discrete memory) than the rock itself.]

We have conducted wave propagation and resonance experiments that partially adhere to, or in some cases do not adhere to, the predictions of the traditional theory. The traditional theory makes no attempt to describe experiments that show hysteresis. We describe a new theory in the following section that provides an explanation of our wave propagation observations, and static observations as well.

The New Paradigm McCall and Guyer (1994) (MG in the following) have introduced a new model of rock elasticity and developed this model in a series of papers. As the model developed, it began to be referred to as the new paradigm. We use this terminology here. [The basic idea behind the model introduced by MG is present in the earlier work of Walsh (1966) and Holcomb (1978;1981).] Let us describe some of the elements of the new paradigm. The new paradigm takes the elastic properties of a macroscopic sample of material to result from the workings of a large number (order 10^{12}) mesoscopic elastic elements in approximately one cubic centimeter of material. These elastic elements individually have complex behavior. The central construct of the new paradigm is P-M space, a space in which the behavior of the elastic elements is tracked, and the density of elastic elements in P-M space, ρ.

One can construct a model P-M space and follow its consequences. This was done by MG. The richness of the new paradigm is that it provides a way of finding ρ, the density of elastic elements in P-M space. Thus the central quantity required for description of stress-strain ,wave propagation, etc., is derived from experiment. The theory is more elaborate than the traditional theory, reduces to the traditional theory in the limit of no hysteresis and no discrete memory, and is also more difficult to apply. The basic approach is outlined here:

(1) Begin with a simple model of the essential features of the elastic elements. This includes a stress-strain relation that may include hysteresis [a stress stain equation of state is a force-displacement relationship. No matter how complex the nature of the individual units, the essential features of its behavior *for the purpose of its participation in a stress-strain response* is the force-displacement rule.]
(2) Create the P-M space description of the workings of a large number of the elastic elements .
(3) Create a model for the geometry of the elastic elements [e.g. place them on a hexagonal lattice].
(4) Prescribe a pressure protocol.
(5) Create a model for calculating the stress strain equation of state of a lattice of elastic elements, e.g. effective medium theory, mean field theory, etc.
(6) Create a stress strain curve.

The procedure for assembly of (1) - (5) into a stress strain equation of state with discrete memory is illustrated in various of the papers (e.g., McCall and Guyer, 1994; Guyer et al., 1995; Guyer et al., 1995) . Let us show the ingredients used in an example. Figures 3a-d will be used for the illustration. These are (a) the behavior of the individual elastic elements, (b) the P-M space density, (c) the pressure protocol and (d) the resulting stress-strain curve.

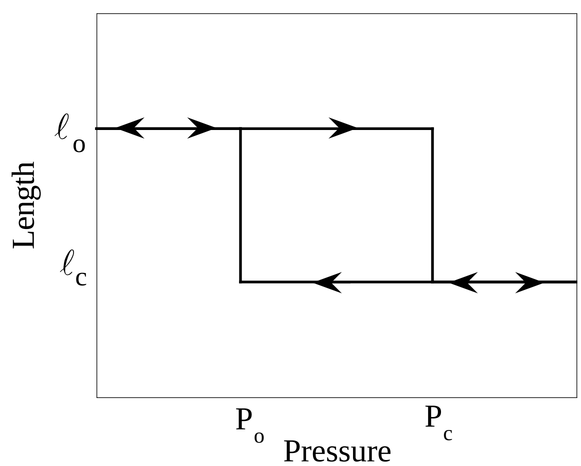


Figure 3a. Hysteretic mesoscopic elastic unit.

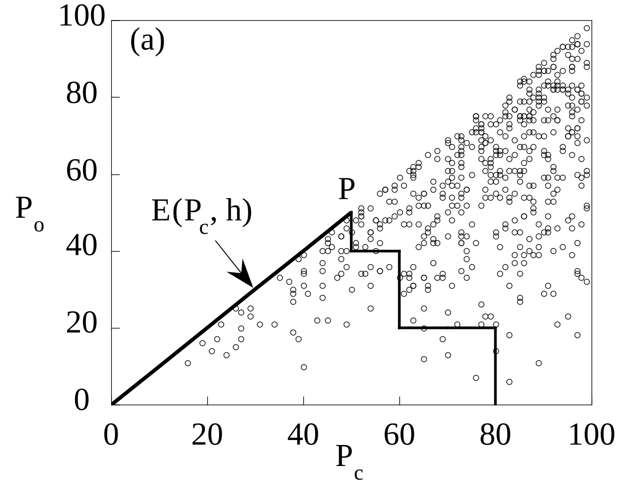


Figure 3b. P-M space for many hysteretic units. See text for explanation.

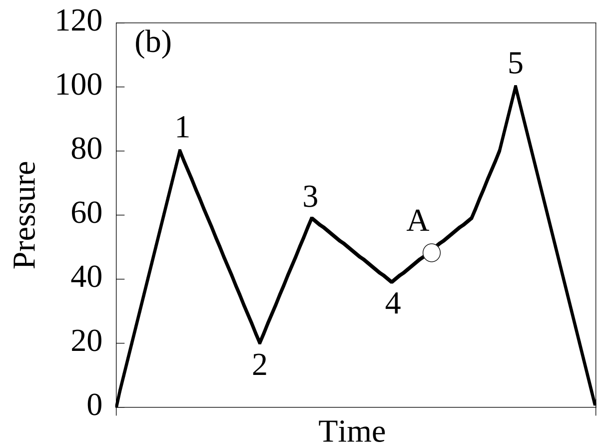


Figure 3c. Pressure protocol.

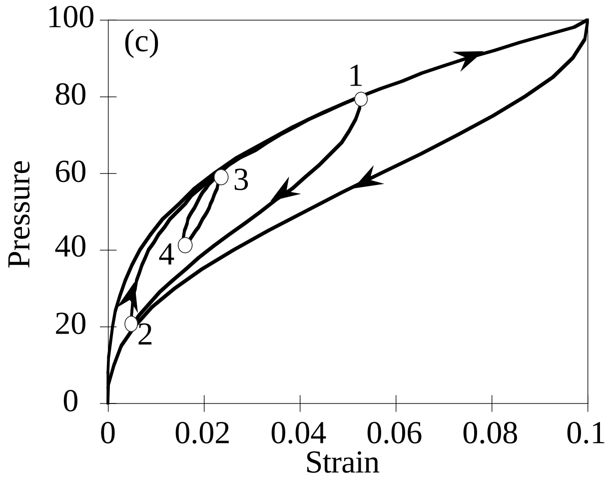


Figure 3d. Stress-strain diagram resulting from hysteretic unit distribution shown in Figure 3b and the pressure protocol shown in Figure 3c.

The basic element of the model, the hysteretic mesoscopic elastic unit, HMEU, is shown in Figure 3a. The figure illustrates length-pressure space for an individual HMEU. In the un-pressured state, the HMEU has length L_o. As pressure is increased to P_c, the unit abruptly shortens, and remains at this length for increasing pressure. As pressure is released, the HMEU abruptly opens at P_c, a pressure lower than or equal to P_o, corresponding to L_c. Note that the HMEU shown in the figure displays hysteresis. A large number of the HMEUs with differing L_o, P_c, P_o, and L_c comprise a model of the compliant portion of a sample material. The HMEUs can be plotted in P-M space. Such a plot is shown in Figure 3b. This plot shows the opening pressure versus closing pressure for all HMEUs in a sample, as illustrated by each circle. The model material is composed of a range of HMEUs, some exhibiting no hysteresis in length-pressure space. The remaining HMEUs exhibit varying ranges of hysteresis. Non-hysteretic units fall on the diagonal and hysteretic units fill out the bottom triangle in P-M space. The more hysteretic the HMEU is, the farther from the diagonal it resides.

The P-M plot is very useful because it can be used to construct a stress-strain plot. This is illustrated as follows. If a pressure protocol such as that shown in Figure 3c is followed, it can be mapped into P-M space as shown by the bold line in Figure 3b.

Figure 3d illustrates the resulting stress-strain relation for the HMEU distribution in Figure 3b coupled with the pressure protocol in Figure 3c. The loop has the all of general characteristics of the data for Berea sandstone shown in Figure 1. In this manner, beginning with the HMEU building block and applying a suite of them in an effective medium approach, a realistic stress-strain plot that includes hysteresis and discrete memory is constructed.

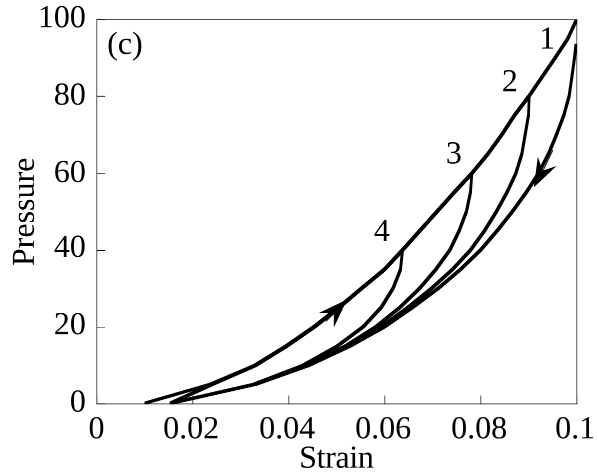
One of the most informative aspects of this model is the insight it provides into the compliant microstructure, the individual HMEUs. For example, in following the pressure protocol shown in Figure 3c, it is observed that as pressure increases in P-M space, HMEUs close below the *horizontal* line along the P_o axis. So at pressure position 1, all units *below* the horizontal line are closed. As pressure is released, those units that re-open are described by a *vertical* line moving toward the origin along the P_c axis. This is important. It means that increasing and decreasing static pressure samples *different volumes of HMEUs*. This is, in fact, the explanation for global hysteresis in Figure 3d..

Another important point is that one can predict the low stress amplitude modulus from the shape of the outer hysteresis loop (see Guyer and McCall, 1994). The model shows that as pressure deviation becomes smaller and smaller, the slope of the tangent of the hysteresis loop (tip to tip) becomes smaller and smaller. There is a continuum of tangent slopes from large static pressure deviations to small. This implies (but remains as yet unproven!) that the dynamic modulus can be predicted from static measurements, and that the explanation of the difference lies in which HMEUs are affected. This will be addressed further below.

We have shown how one begins with a suite of HMEUs with no assumption about their properties except their opening and closing lengths and their distribution in P-M space, and produced a hysteretic stress-strain curve. Guyer and McCall also show that one can begin with the stress-strain relation for a rock, and invert the data for the HMEU structure with no assumptions about the details (e.g., crack structure, fluid content). The characteristics of these details is a natural step and are part of the work proposed here. This approach allows one to interrogate the medium with no assumptions about physical characteristics, and then infer them. The opposite approach as has been used by many (e.g., Toksoz et al., 1976; Walsh, 1965; O'Connell and Budiansky, 1977)

The new paradigm gives the density in P-M space central status. The P-M space density takes the place of the nonlinear parameters of the traditional theory. In the illustration above the density was assumed known. An important feature of the new paradigm is that it suggests a procedure for learning the density in P-M space from a suitable data set. This suggestion has been implemented and is described in the paper by Guyer et al. (1995). An illustration of what results from this work is given in Figure 4 where we show a P-M space density found from analysis of a stress strain equation of state on a Berea sandstone. One obtains the distribution of the HMEUs from the inversion. Note the concentration along the diagonal and at small Po and Pc. In addition, the resulting P-M space density has been used to successfully predict the result of additional stress strain experiments.

Thus far we have devoted our discussion of the model to the static case, i.e. we can describes stress-strain data (experiments involving static or quasi-static stress-strain manipulation.). Now let us turn to discussion of waves, i.e. dynamic stress-strain experiments.



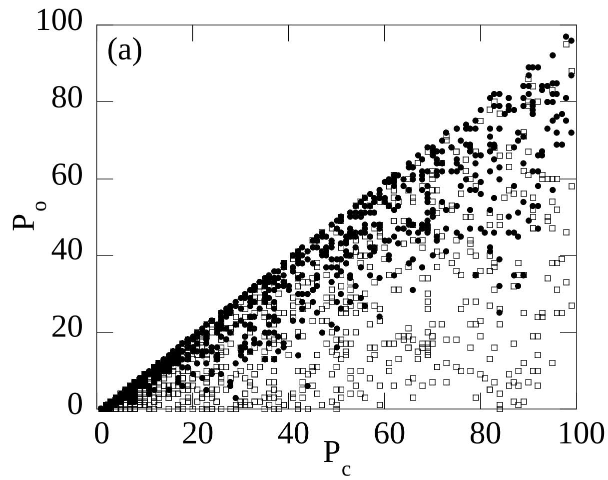


Figure 4. Measured stress-strain curve for Berea sandstone (left) and P-M space representation of the stress-strain data (right).

Wave Propagation. Does the P-M space model describe observations from wave propagation as well? We believe that it does. We describe a measurement in sandstone as an example. Figure 5 shows the result of a resonant bar experiment using a sample of the "linear" material, PVC, and a sample of highly elastically nonlinear material, Fontainebleau sandstone. In this experiment the frequency is swept through the fundamental resonance at fixed drive amplitude, for a series of increasing drive amplitudes. As the drive amplitude increases the resonance frequency remains constant in the PVC; however, in the sandstone, the resonant frequency shifts to lower frequency with increasing drive level. The shift in resonance frequency is proportional to the amplitude of the drive. The peak shift in the sandstone is a clear manifestation of elastic nonlinear response, a response that the PVC does not display. The behavior in the sandstone is different for downgoing versus upgoing frequency sweep. This is typical of rock, but beyond the scope of our focus here.

Application of the traditional theory to this experimental scenario leads to results in qualitative and quantitative disagreement with the experiment (Guyer et al., 1995). For example, prediction of the change in resonant frequency with detected acceleration is entirely incorrect by application of the traditional theory. This experiment can be given quantitative explanation using the new paradigm (Guyer et al., 1995). The explanation is both qualitative and quantitative. The quantitative explanation is gratifying. It is achieved using, as input, data from a P-M space density found empirically.

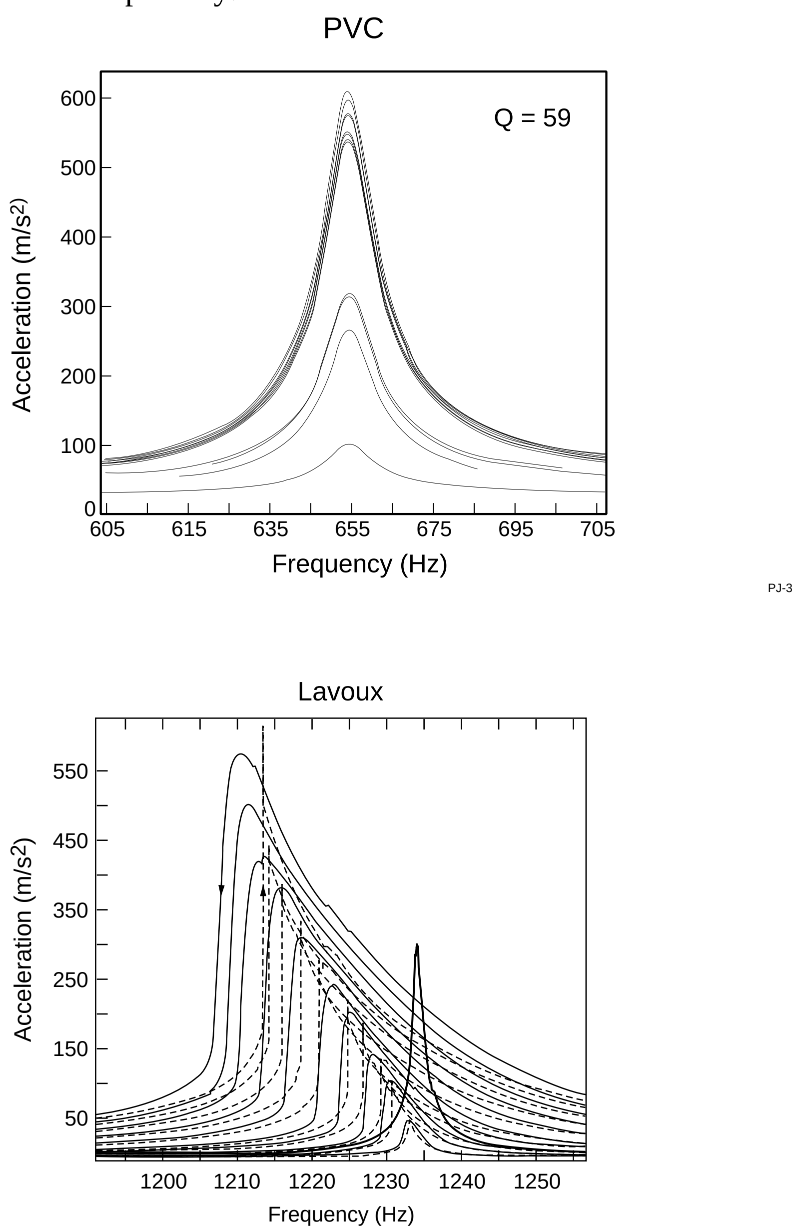


Figure 6. Resonant response as function of drive level in PVC (left) and sandstone (right).

We have in hand a powerful theoretical construct for the synthesis and use of data on rocks and other compliant materials. We also have a variety of experimental tools, stress strain apparatus, wave propagation apparatus, and resonant bar apparatus, to bring to bear. Our goal is an integrated experimental/theoretical program to push the understanding of rock elasticity significantly forward. The first step toward this goal is the development of a robust quantitative methods for measuring and applying the equation of state.